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FIRST Semester B. E. Degree Semester End Examination (SEE), Jan/ Feb 2024

Calculus, Differential Equations and Linear Algebra

(Model Question Paper - 1)

[Time: 3 Hours] **[Maximum Marks: 100]**

Instructions to students:

i. **Answer FIVE FULL Questions as per choice.**
ii. **Use BLACK ball point pen for text, figure, table, etc.**

		Module-1	Marks	CO	RBT Level
1.	**a)**	With usual notation, prove that $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2$	6	L2	CO1
	b)	Using Maclaurin's expansion series prove that $\sqrt{1+\sin 2x}=1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}---$	7	L2	CO1
	c)	Find the radius of curvature of the curve $x^3+y^3 = 3axy$ at the point (3a/2,3a/2)	7	L2	CO1
		OR			
2.	**a)**	Obtain the pedal equation of the curve $r=a(1-\cos\theta)$ and hence show that $\rho=(2/3)\sqrt{2ar}$	6	L2	CO1
	b)	$x+y+z=u, y+z=v, z=uvw,$ find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	7	L2	CO1
	c)	Examine the following function for extreme values of the function $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$	7	L2	CO1
		Module-2			
3.	**a)**	Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$	6	L2	CO2
	b)	Evaluate $\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$ by changing the order of integration	7	L2	CO2
	c)	$\text{Prove that } \beta(m,n)=\frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$	7	L2	CO2
		OR			
4.	**a)**	Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dy\,dx$ by changing in to polar coordinates	6	L2	CO2

	b)	Find the area between the parabolas $y^2 = 4ax$ & $x^2 = 4ay$	7	L2	CO2
	c)	Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}.d\theta = \pi$	7	L2	CO2
		Module-3			
5.	**a)**	Show that $\vec{F}$=(6xy+z^3)I+(3x^2-z)J-(3xz^2-y)K is irrotational, find ϕ such that $F = \nabla\phi$.	6	L2	CO3
	b)	Find the angle between surfaces $x^2 + y^2 + z^2 = 9$ & $x^2 + y^2 - 3 = x$ at (2,-1,2)	7	L2	CO3
	c)	Find the scale factors of cylindrical system	7	L2	CO3
		OR			
6.	**a)**	Find Curl (Curl$\vec{F}$) where $\vec{F}=xy\hat{\imath} + y^2z\hat{\jmath} + z^2y\hat{k}$	6	L2	CO3
	b)	Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at (-1,-2,-1) in the direction of the vector $2\hat{\imath}$-$\hat{\jmath}$-$2\hat{k}$	7	L2	CO3
	c)	Prove that cylindrical system is orthogonal.	7	L2	CO3
		Module-4			
7.	**a)**	Solve $\frac{dy}{dx} + xsin2y = x^3cos^2$y	6	L2	CO4
	b)	Given $y = ke^{-2x} + 3x$, find the member of its orthogonal trajectories passing through the point (0,3)	7	L2	CO4
	c)	Solve $\left(D^2+2D+4\right)y=2x^2+3e^{-x}$	7	L2	CO4
		OR			
8.	**a)**	*Solve* $\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$	6	L2	CO4
	b)	*Solve* $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$ *given* $y=0, y'=3$ *for* $x=0$	7	L2	CO4
	c)	By the method of variation of parameters solve $y''+4y=\tan 2x$	7	L2	CO4
		Module-5			
9.	**a)**	Reduce the matrix into its normal form and hence find its rank $A=\begin{bmatrix}1&2&4&3\\2&4&6&8\\4&8&12&16\\1&2&3&4\end{bmatrix}$.	6	L2	CO5
	b)	Solve the system of equations by Gauss siedel method: 2x + 5y + 7z = 52, 2x + y – z = 0, x + y + z = 9.	7	L2	CO5
	c)	Find the largest eigen value and the corresponding eigen vector of the matrix	7	L2	CO5

$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method, use $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as initial vector, take five iterations.

OR

10.	**a)**	Find the rank of the matrix $\begin{bmatrix} 91 & 92 & 93 & 94 & 95 \\ 92 & 93 & 94 & 95 & 96 \\ 93 & 94 & 95 & 96 & 97 \\ 94 & 95 & 96 & 97 & 98 \\ 95 & 96 & 97 & 98 & 99 \end{bmatrix}$	6	L2	CO5
	b)	For what values of λ and μ the system of equations 2x+3y+5z = 9, 7x+3y-2z = 8, 2x+3y+λz = μ has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.	7	L2	CO5
	c)	Solve the following system of equations by Gauss-Elimination method: $x+y+z=8;\ -x-y+2z=-4;\ 3x+5y-7z=14$	7	L2	CO5

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