**23MAT11C**

| USN | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|

# FIRST Semester B. E. Degree Semester End Examination (SEE), Jan/ Feb 2024

# Calculus and Linear Algebra

**(Model Question Paper - 1)**

| **[Time: 3 Hours]** | | **[Maximum Marks: 100]** |
|---|---|---|

**<u>Instructions to students</u>:**

i. **Answer FIVE FULL Questions as per choice.**
ii. **Use BLACK ball point pen for text, figure, table, etc.**

| | | **Module-1** | **Marks** | **CO** | **RBT Level** |
|---|---|---|---|---|---|
| **1.** | **a)** | Find the angle between radius vector and tangent at the point on the curve. | 6 | L2 | CO1 |
| | **b)** | Find the angle between the radius vector and the tangent and also find the slope of the tangent for the curve $r\cos^2\left(\frac{\theta}{2}\right)=a$ at $\theta=\frac{2\pi}{3}$ | 7 | L2 | CO1 |
| | **c)** | Find the radius of curvature for the curve $x^3+y^3=3axy$ at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ | 7 | L2 | CO1 |
| | | **OR** | | | |
| **2.** | **a)** | Show that the curve $r^n\cos n\theta=a^n$ and $r^n\sin n\theta=b^n$ intersect orthogonally. | 6 | L2 | CO1 |
| | **b)** | Find the pedal equation for the curve $r=2a/1+\cos\theta$. | 7 | L2 | CO1 |
| | **c)** | Evaluate $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$ | 7 | L2 | CO1 |
| | | **Module-2** | | | |
| **3.** | **a)** | Using Maclaurin's series, prove that $\sqrt{1+\sin 2x}=1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}\ldots$ | 6 | L2 | CO2 |
| | **b)** | If $z=e^x(x\sin y+y\cos y)$, prove that $u_{xx}+u_{yy}=0$ | 7 | L2 | CO2 |
| | **c)** | Find the extreme values of the function $x^3+3xy^2-15x^2-15y^2+72x$ | 7 | L2 | CO2 |
| | | **OR** | | | |
| **4.** | **a)** | Expand $\tan^{-1}x$ in the powers of $(x-1)$ upto the terms containing fourth degree. | 6 | L2 | CO2 |
| | **b)** | If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$ | 7 | L2 | CO2 |
| | **c)** | If $u=\frac{yz}{x}, v=\frac{xz}{y},\ w=\frac{xy}{z}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. | 7 | L2 | CO2 |
| | | **Module-3** | | | |
| **5.** | **a)** | $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dydxdz$ | 6 | L2 | CO3 |

| | | | | | |
|---|---|---|---|---|---|
| | b) | Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ by changing the order of integration. | 7 | L2 | CO3 |
| | c) | Obtain the relationship between beta and gamma functions. | 7 | L2 | CO3 |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| 6. | a) | Change the integral $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ into polars and hence evaluate. | 6 | L2 | CO3 |
| | b) | Show that $\int_0^\infty \sqrt{y}e^{-y^2} dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$ | 7 | L2 | CO3 |
| | c) | Find by double integration the area enclosed by the curve $r = a(1+\cos\theta)$ between $\theta = 0$ and $\theta = \pi$. | 7 | L2 | CO3 |

**Module-4**

| | | | | | |
|---|---|---|---|---|---|
| 7. | a) | Solve $\frac{dy}{dx} - \frac{1}{2}\left(1+\frac{1}{x}\right)y + \frac{3y^3}{x} = 0$ | 6 | L2 | CO4 |
| | b) | Solve $xy\left(\frac{dy}{dx}\right)^2 - \left(x^2+y^2\right)\frac{dy}{dx} + xy = 0$ | 7 | L2 | CO4 |
| | c) | A body in air at $25^0$C cools from $100^0$C to $75^0$C in 1 minute. Find the temperature of the body at the end of 3 minutes. | 7 | L2 | CO4 |

**OR**

| | | | | | |
|---|---|---|---|---|---|
| 8. | a) | Solve $\left(y^3 - 3x^2y\right)dx - \left(x^3 - 3xy^2\right)dy = 0$. | 6 | L2 | CO4 |
| | b) | Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$, where $\lambda$ is the parameter. | 7 | L2 | CO4 |
| | c) | Solve the equation $(px-y)(py+x) = a^2p$, by taking $X = x^2$, $Y = y^2$. | 7 | L2 | CO4 |

**Module-5**

| | | | | | |
|---|---|---|---|---|---|
| 9. | a) | Determine the rank of the matrix $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. | 6 | L2 | CO5 |
| | b) | Solve the system of equations by Gauss elimination method $2x+y+4z=12, 4x+11y-z=33, 8x-3y+2z=20.$ | 7 | L2 | CO5 |
| | c) | Find the largest eigen value and the corresponding eigen vector of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by power method, use $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as initial vector, take five iterations. | 7 | L2 | CO5 |

**OR**

| | | | Marks | Level | CO |
|---|---|---|---|---|---|
| **10.** | **a)** | Reduce the matrix into its normal form and hence find its rank $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$. | 6 | L2 | CO5 |
| | **b)** | Solve the system of equations by Gauss-Seidel method $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25.$ | 7 | L2 | CO5 |
| | **c)** | For what values of λ and μ the system of equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu.$ has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. | 7 | L2 | CO5 |

*********