USN										

FIRST Semester B. E. Degree Semester End Examination (SEE), Jan/ Feb 2024

Fundamentals of Infinite series, Calculus & Linear Algebra

(Model Question Paper - 1)

[Time: 3 Hours] **[Maximum Marks: 100]**

Instructions to students:

i. **Answer FIVE FULL Questions as per choice.**
ii. **Use BLACK ball point pen for text, figure, table, etc.**

		Module-1	Marks	CO	RBT Level
1.	**a)**	Test the convergence of the series $\sum_{n=1}^{\infty}\sqrt{\frac{3^n-1}{2^n+1}}$.	6	L2	CO1
	b)	Discuss the convergence of the series $\frac{1}{\log 2}-\frac{1}{\log 3}+\frac{1}{\log 4}-\frac{1}{\log 5}+\ldots$ by using Alternative series.	7	L2	CO1
	c)	Expand $\tan^{-1} x$ in powers of (x-1) up to the term containing fourth degree.	7	L2	CO1
		OR			
2.	**a)**	Find the values of x for the series convergence: $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\infty$.	6	L2	CO1
	b)	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n!}{(n^n)^2}$ by using D' Alembert ratio test.	7	L2	CO1
	c)	Expand $\log(1+x)$ by Maclaurin's series upto the term containing x^4.	7	L2	CO1
		Module-2			
3.	**a)**	With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$.	6	L2	CO2
	b)	Find the pedal equation for the curve $r^n = a^n(\sin n\theta + \cos n\theta)$.	7	L2	CO2
	c)	Show that the radius of curvature at any point of the cardioid $r = a(1-\cos\theta)$ varies as $\sqrt{r}$.	7	L2	CO2
		OR			
4.	**a)**	Show that the curve $r = a(1+cos\theta)$, and $r = b(1-cos\theta)$ intersect at right angle.	6	L2	CO2
	b)	Derive the expression for radius of curvature in the case of a Cartesian curve.	7	L2	CO2
	c)	Evaluate i) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{3}}$ ii) $\lim_{x\to\frac{\pi}{2}}(\cos x)^{\frac{\pi}{2}-x}$	7	L2	CO2
		Module-3			

			Marks	Level	CO
5.	a)	If z = $f(x+ct)+g(x-ct)$ prove that $\frac{\partial^2 z}{\partial t^2}=c^2\frac{\partial^2 z}{\partial x^2}$.	6	L2	CO3
	b)	If $u=\frac{yz}{x}, v=\frac{xz}{y}, w=\frac{xy}{z}$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	7	L2	CO3
	c)	If *x, y, z* are the angles of a triangle show that the maximum value of $\cos x \cdot \cos y \cdot \cos z$ is 1/8.	7	L2	CO3
		OR			
6.	a)	If $u=f(x-y,y-z,z-x)$ then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$=0.	6	L2	CO3
	b)	Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem.	7	L2	CO3
	c)	Find the extreme values of the function, $f(x,y)=x^3+y^3-3x-12y+20$.	7	L2	CO3
		Module-4			
7.	a)	Solve $x\frac{dy}{dx}+y=x^3y^6$.	6	L2	CO4
	b)	Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$, where λ is the parameter.	7	L2	CO4
	c)	Show that the differential equation for the current *i* in an electrical current containing an inductance *L* and a resistance *R* in series and acted on by an electromotive force $E\sin(\omega t)$ satisfies the equation $L\frac{di}{dt}+Ri=E\sin(\omega t)$. Find the value of the current at any time *t*, if initially there is no current in the circuit.	7	L2	CO4
		OR			
8.	a)	Solve $(1+2xycosx^2-2xy)dx+(sinx^2-x^2)dy=0$.	6	L2	CO4
	b)	A body originally at 80^{0}C cools down to 60⁰C in 20 minutes, the temperature of the air being 40⁰C. What will be temperature of the body after 40 minutes from the original?	7	L2	CO4
	c)	Solve $(px-y)(py+x)=2p$.	7	L2	CO4
		Module-5			
9.	a)	Determine the rank of the matrix $\begin{bmatrix}0&1&-3&-1\\1&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$.	6	L2	CO5
	b)	Solve the system of equations by Gauss elimination method x +4y-z = -5, x+y-6z =-12, 3x-y-z = 4.	7	L2	CO5
	c)	Find the largest eigen value and the corresponding eigen vector of the matrix A = $\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$ by power method, use $\begin{bmatrix}1\\0\\0\end{bmatrix}$ as initial vector, take five iterations.	7	L2	CO5

OR

10.	**a)**	Reduce the matrix into its normal form and hence find its rank $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.	6	L2	CO5
	b)	For what values of λ and μ the system of equations 2x+3y+5z = 9, 7x+3y-2z = 8, 2x+3y+λz = μ has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.	7	L2	CO5
	c)	Solve the system of equations by Gauss-Seidel method x+y+54z = 110, 27x+6y-z = 85, 6x+15y+2z = 72.	7	L2	CO5

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