USN										

FIRST Semester B. E. Degree Semester End Examination (SEE), Jan/ Feb 2024

Advanced Calculus

(Model Question Paper - 1)

[Time: 3 Hours]		[Maximum Marks: 100]
	Instructions to students: i. Answer FIVE FULL Questions as per choice. ii. Use BLACK ball point pen for text, figure, table, etc.	

		Module-1	Marks	CO	RBT Level
1.	a)	Find the length of the perpendicular from the pole to the tangent.	6	L2	CO1
	b)	Find the angle between the radius vector and the tangent and also find the slope of the tangent for the curve $\frac{2a}{r} = (1 - \cos\theta)$ at $\theta = \frac{2\pi}{3}$	7	L2	CO1
	c)	Find the radius of curvature for the curve $y = ax^2 + bx + c$ at $x = \frac{1}{2a}\left[\sqrt{a^2 - 1} - b\right]$	7	L2	CO1
		OR			
2.	a)	Show that the curve $r = a(1 + cos\theta)$ and $r^2 = a^2cos2\theta$ intersect orthogonally.	6	L2	CO1
	b)	Find the pedal equation for the curve $r^n = a^n \cos n\theta$.	7	L2	CO1
	c)	Evaluate $\lim_{x \to 0} \frac{tanx - x}{x^2\, tanx}$	7	L2	CO1
		Module-2			
3.	a)	Using Maclaurin's series, expand $\log(1 + \sin x)$ in powers of x upto the term x^4.	6	L2	CO2
	b)	If $u = \tan^{-1}(\frac{y}{x})$, verify that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$	7	L2	CO2
	c)	Find the extreme values of the function $xy(a - x - y)$.	7	L2	CO2
		OR			
4.	a)	Expand $\log(\sec x)$ upto the term containing x^6 using Maclaurin's series.	6	L2	CO2
	b)	If $u = f(x - y, y - z, z - x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	7	L2	CO2
	c)	If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	7	L2	CO2
		Module-3			
5.	a)	Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$.	6	L2	CO3
	b)	Evaluate $\int_0^1 \int_x^1 \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$. by changing the order of integration.	7	L2	CO3
	c)	Obtain the relationship between beta and gamma functions.	7	L2	CO3
		OR			
6.	a)	Change the integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} y\sqrt{x^2 + y^2}\, dy\, dx$ into polars and hence evaluate.	6	L2	CO3

			Marks	Level	CO
	b)	Show that $\int_0^\infty \sqrt{y}e^{-y^2}dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}dy = \frac{\pi}{2\sqrt{2}}$	7	L2	CO3
	c)	Find the volume generated by the revolution of the cardioide $r = a(1+\cos\theta)$ about the initial line.	7	L2	CO3
		Module-4			
7.	a)	Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$	6	L2	CO4
	b)	Find the orthogonal trajectories of the family of astroids $x^{2/3} + y^{2/3} = a^{2/3}$.	7	L2	CO4
	c)	Solve $xy\left(\frac{dy}{dx}\right)^2 - \left(x^2+y^2\right)\frac{dy}{dx} + xy = 0$	7	L2	CO4
		OR			
8.	a)	Solve $\frac{dy}{dx} + \frac{x+3y-4}{3x+9y-2} = 0$.	6	L2	CO4
	b)	A body in air at 25^0C cools from 100^0C to 75^0C in 1 minute. Find trhe temperature of the body at the end of 3 minutes.	7	L2	CO4
	c)	Solve the equation $(px-y)(py+x) = a^2p$, by taking $X = x^2$, $Y = y^2$.	7	L2	CO4
		Module-5			
9.	a)	Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.	6	L2	CO5
	b)	Solve the system of equations by Gauss elimination method x +y+z = 9, x-2y+3z =8, 2x+y-z = 3.	7	L2	CO5
	c)	Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by power method, use $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as initial vector.	7	L2	CO5
		OR			
10.	a)	Reduce the matrix into its normal form and hence find its rank $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \\ 2 & 3 & 1 & 4 \end{bmatrix}$.	6	L2	CO5
	b)	For what values of λ and μ the system of equations 2x+3y+5z = 9, 7x+3y-2z = 8, 2x+3y+λz = μ has (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.	7	L2	CO5
	c)	Solve the system of equations by Gauss-Seidel method 20x+y-2z = 17, 3x+20y-z = -18, 2x-3y+20z = 25.	7	L2	CO5