Power System Analysis-2 (18EE71) - Odd Sem 2021-22

Solution to Question Bank

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Power System Analysis-2

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Module 1

1. a) Define the following and give an illustrative example: i) tree and co-tree ii) Basic loops iii) Basic cut sets iv) primitive network v) Bus frame of reference.

June 2015, Dec 2016, Dec 2015, June 2017

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph.* A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph : This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:
e = number of elements = 6
n = number of nodes = 4
b = number of branches = n-1 = 3
l = number of links = e-b = 3
Tree = T(1,2,3) and
Co-tree = T(4,5,6)

Sub-graph :sG is a sub-graph of G if the following conditions are satisfied:
sG is itself a graph
Every node of sG is also a node of G
Every branch of sG is a branch of G
For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

Loop :A sub-graph L of a graph G is a loop if
L is a connected sub-graph of G

Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, althoughthe set(1,3,4,5) is a valid loop. The KVL (Kirchhoff‟s Voltage Law) for the loop isstated as follows: In any lumped network, the algebraic sum of the branch voltagesaround any of the loops is zero.

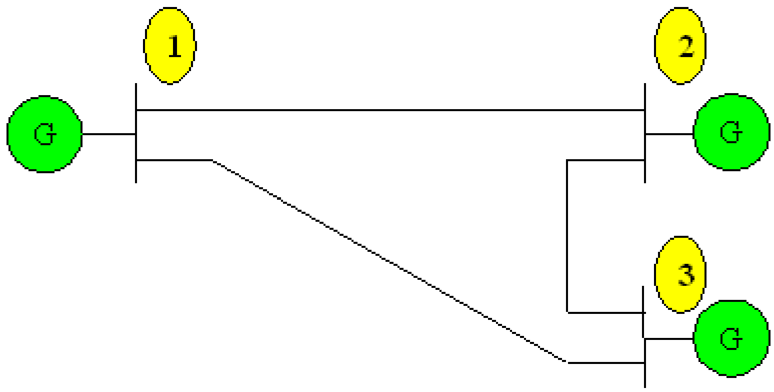


Fig 1a. Single line diagram of a power system

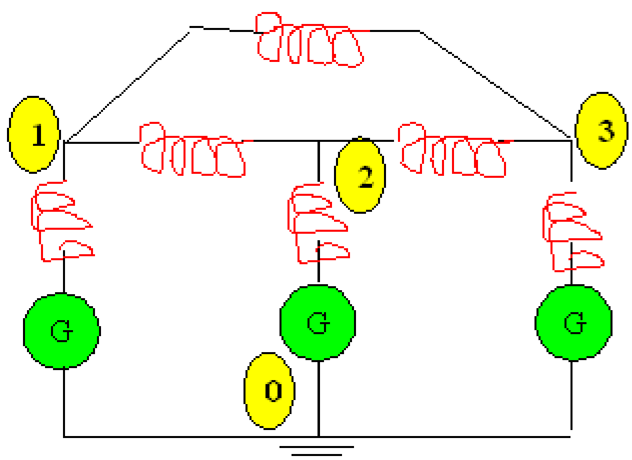


Fig 1b. Reactance diagram

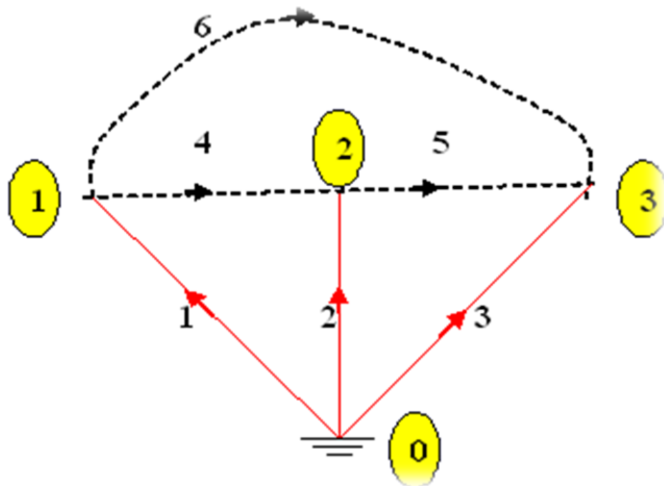


Fig 1c. Oriented Graph

Cutset : It is a set of branches of a connected graph G which satisfies the following conditions :

The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.

The removal of all but one of the branches of the set, leaves the remaining graph connected. Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff‟s Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1 **(1)**
For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: $\boldsymbol{l}$ = e – b = e – n + 1 **(2)**

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets*or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

2. Derive an expression for obtaining Y-bus using singular transformations.
June 2016,**Dec 2016, Dec.2015, June 2017**

In the bus frame of reference, the performance of the interconnected network is described by *n*independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).
For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

IBUS = YBUS EBUS
Where EBUS = vector of bus voltages measured with respect to reference bus
IBUS = Vector of currents injected into the bus
YBUS = bus admittance matrix
The performance equation of the primitive network in admittance form is given by
i + j = [y] v
Pre-multiplying by At (transpose of A), we obtain

At i +At j = At [y] v
At i =0,
since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff"s law is zero. Similarly, At j gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

At j = IBUS
we have, IBUS = At [y] v
However, we have

v =A EBUS
And hence substituting in equation we get,

IBUS = At [y] A EBUS
we obtain,

YBUS = At [y] A
The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

ZBUS = $YBUS^{-1}$
Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

3. Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: Z1=Z2=0.2, Z3=0.25, Z4=Z5=0.1 and Z6=0.4 units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Dec 2016

Solution:
The element node incidence matrix, *Aˆ* can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ =

1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	0	1	-1
0	1	0	-1

Based on the conventional definitions of the elements of $A\hat{}$, the oriented graph can be formed as under:

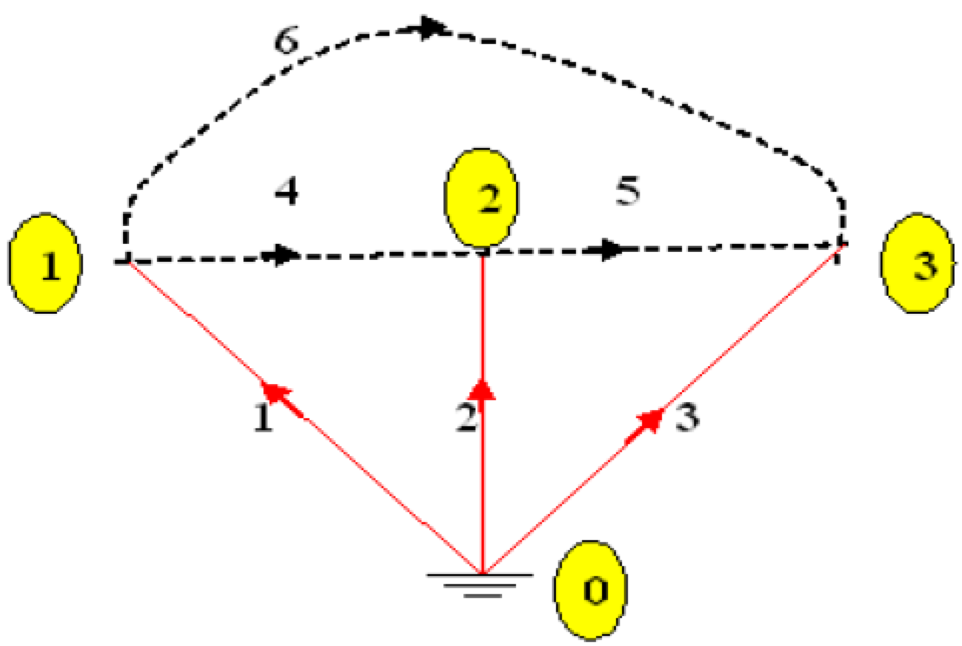


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z] =

0.2	0	0	0	0	0
0	**0.2**	0	0	0	0
0	0	**0.25**	0	0	0
0	0	0	**0.1**	0	0
0	0	0	0	**0.1**	0
0	0	0	0	0	**0.4**

And

[y] =

5.0	0	0	0	0	0
0	**5.0**	0	0	0	0
0	0	**4.0**	0	0	0
0	0	0	**10**	0	0
0	0	0	0	**10**	0
0	0	0	0	0	**2.5**

4. **What is a primitive network? Give the representation of a typical component and arrive at the performance equations both in impedance and admittance forms.**

Dec.2015,June 2016, June 2017

PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

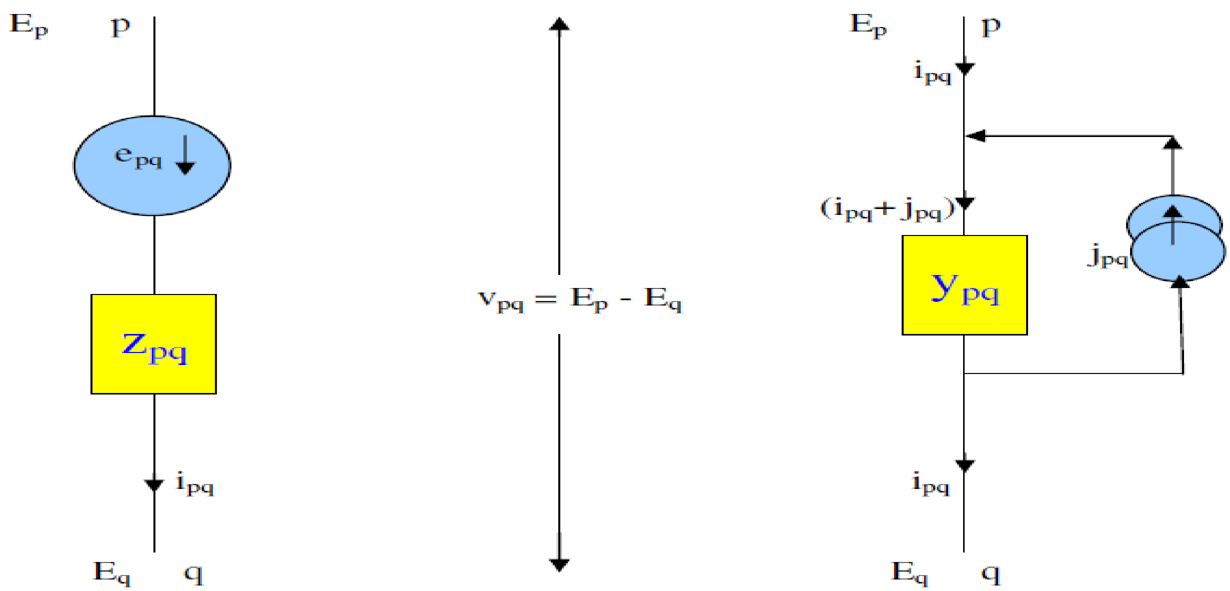


Fig.2 Representation of a primitive network element (a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,
vpq = voltage across the element p-q,
epq = source voltage in series with the element p-q,
ipq= current through the element p-q,
jpq= source current in shunt with the element p-q,
zpq= self impedance of the element p-q and
ypq= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, Vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

vpq + epq = zpqipq (in its impedance form)
ipq + jpq = ypqvpq (in its admittance form)

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:
jpq = - ypqepq

A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$v + e = [z]\, i$

$i + j = [y]\, v$

Primitive network matrices:

A diagonal element in the matrices, [z] or [y] is the self impedancezpq-pq or self admittance, ypq-pq. An off-diagonal element is the mutual impedance, zpq-rs or mutual admittance, ypq-rs, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

5. **For the sample network-oriented graph shown in Fig.below by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and Aˆ . Also show the partitioned form of the matrix-A.**

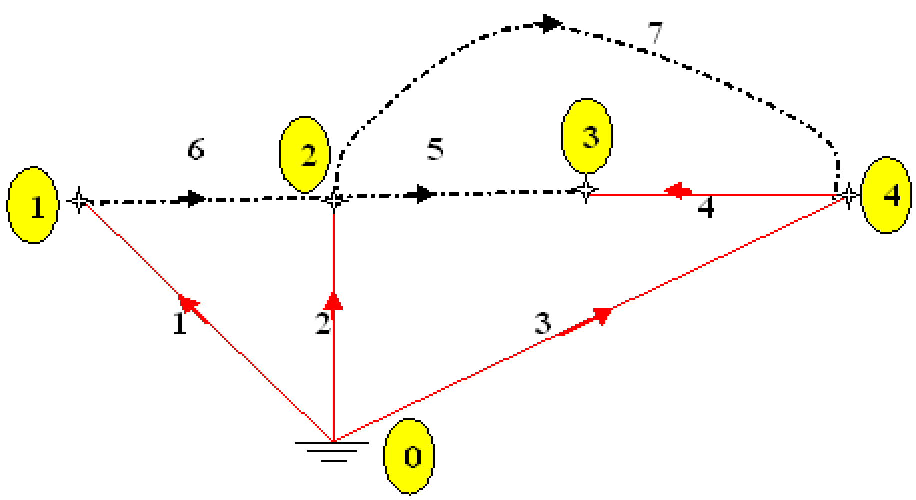


Fig. Sample Network-Oriented Graph

June 2017,June 2015, Dec 2015, June 2016

$$\hat{A} = \text{Elements} \overset{\text{nodes}}{\begin{bmatrix} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

$$\mathbf{A} = \text{Elements} \overset{\text{buses}}{\begin{bmatrix} e\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$$A_b = \text{branches}\overset{\text{buses}}{\begin{bmatrix} b\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}}$$

$$A_l = \text{links}\overset{\text{buses}}{\begin{bmatrix} l\backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

6. For the sample-system shown in Fig. E3, obtain an oriented graph. Byselecting a tree, T(1,2,3,4), obtain the incidence matrices A andAˆ . Also show thepartitioned form of the matrix-A.

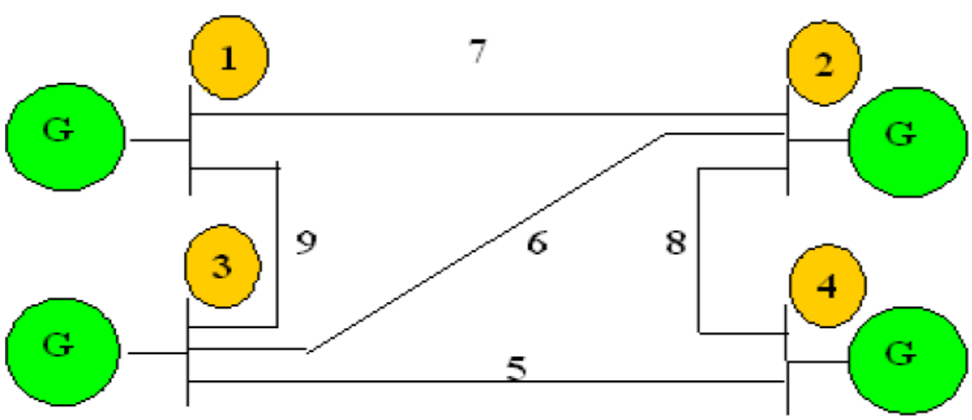


Fig. E3a. Sample Example network

June 2017, June 2015

Consider the oriented graph of the given system as shown in figure E3b, below.

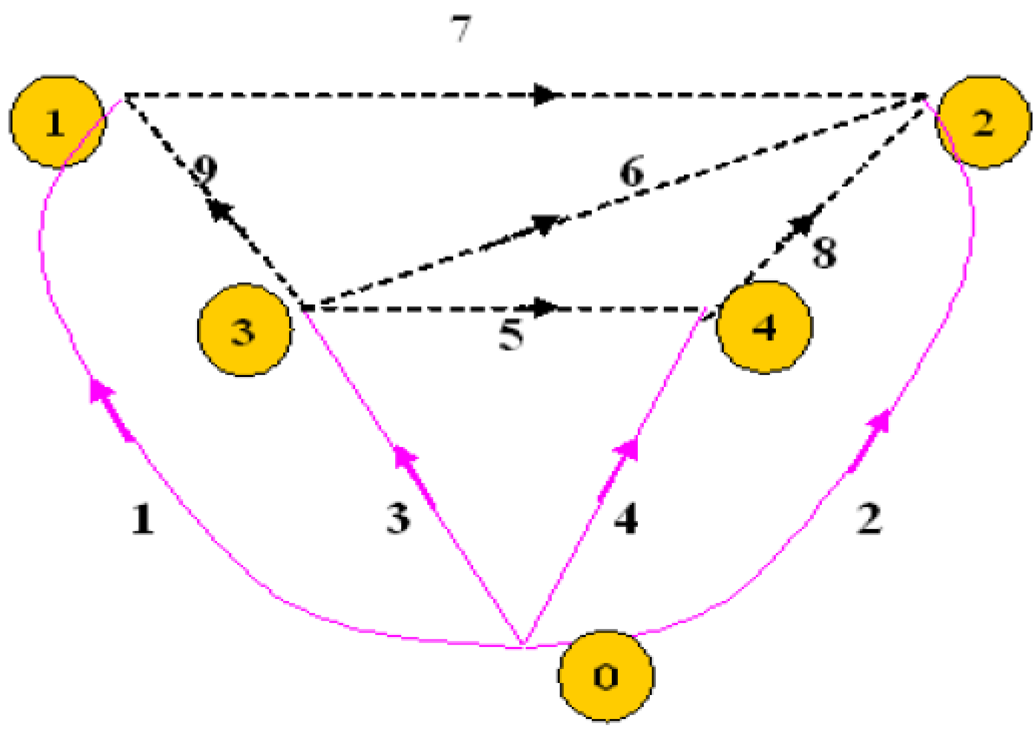


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices • and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$A_b =$

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$A_l =$

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

7. **For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table. June 2016**

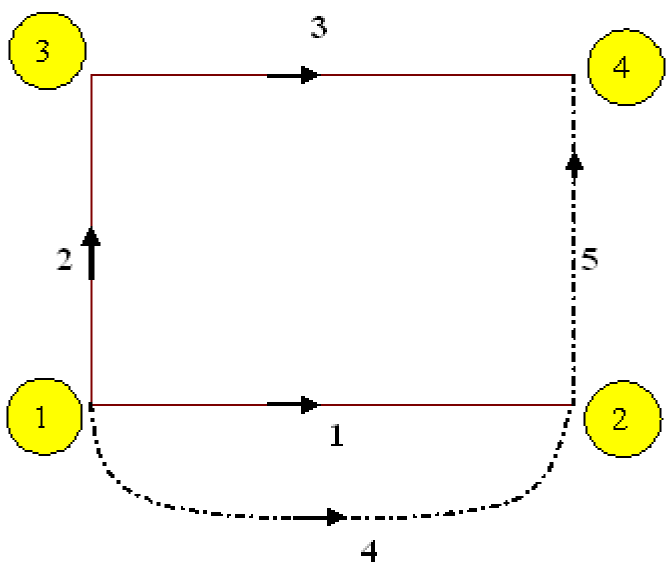


Fig System

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by

$$[z]=\begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix [y] = [z]-1 and given by,

$$[y]=\begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

8. Derive the expression for Y_{bus} using Inspection method. June 2015,June 2016

Consider the 3-node admittance network as shown in figure5. Using the basic branchrelation: I = (YV), for all the elemental currents and applying Kirchhoff‟s CurrentLaw principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)
At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1)
At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2)

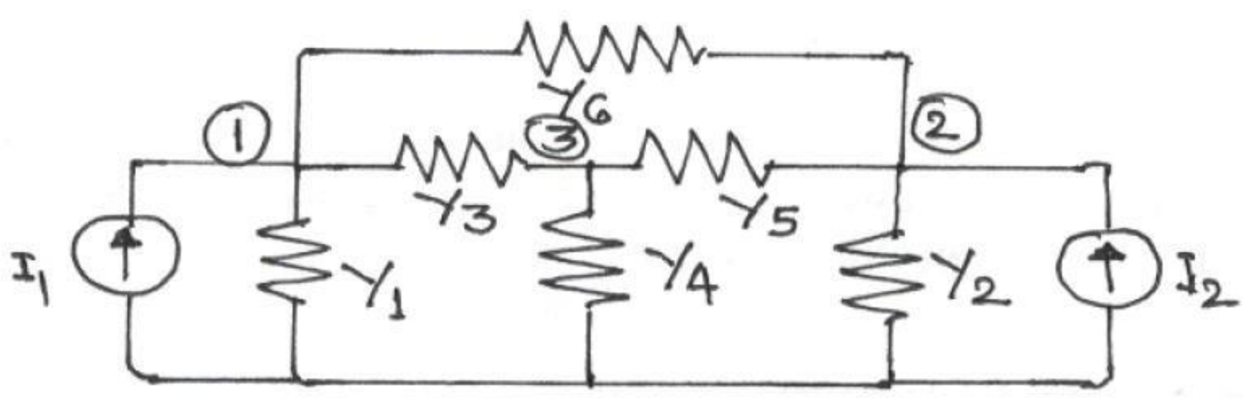


Fig.Example System for finding YBUS

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

In other words, the relation of equation (9) can be represented in the form
IBUS = YBUS EBUS
Where, YBUS is the bus admittance matrix, IBUS & EBUS are the bus current and busvoltage vectors respectively.By observing the elements of the bus admittance matrix, YBUS of equation (13), it isobserved that the matrix elements can as well be obtained by a simple inspection ofthe given system diagram:
Diagonal elements: A diagonal element (Yii) of the bus admittance matrix,YBUS, is equal to the sum total of the admittance values of all the elementsincident at the bus/node i,
Off Diagonal elements: An off-diagonal element (Yij) of the bus admittancematrix, YBUS, is equal to the negative of the admittance value of theconnecting element present between the buses I and j, if any.This is the principle of the rule of inspection. Thus the algorithmic equations for therule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)
Yij = - yij (j = 1,2,…….n)

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of elementconnected between buses i and j and yii is the admittance of element connectedbetween bus i and ground (reference bus).

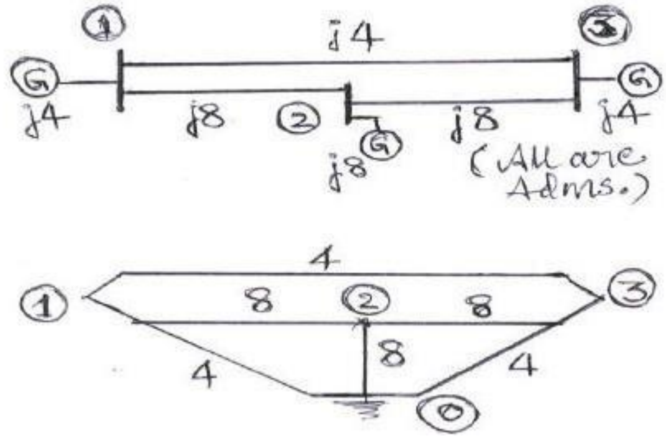
Bus impedance matrix

In cases where, the bus impedance matrix is also required, it cannot be formed bydirect inspection of the given system diagram. However, the bus admittance matrixdetermined by the rule of inspection following the steps explained above, can beinverted to obtain the bus impedance matrix, since the two matrices are interinvertible.
Note: It is to be noted that the rule of inspection can be applied only to those powersystems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example : Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



1. Obtain the general expressions for Z_{bus} building algorithm when a branch is added to the partial network.

June 2016

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector ypq-rs is not equal to zero and **Zij= Zji**" i,j=1,2,…m,q

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i ....p,….m, q
(13)
Hence, Eq = Zqi ;Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i……..p,….m, _q
(14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix}$$

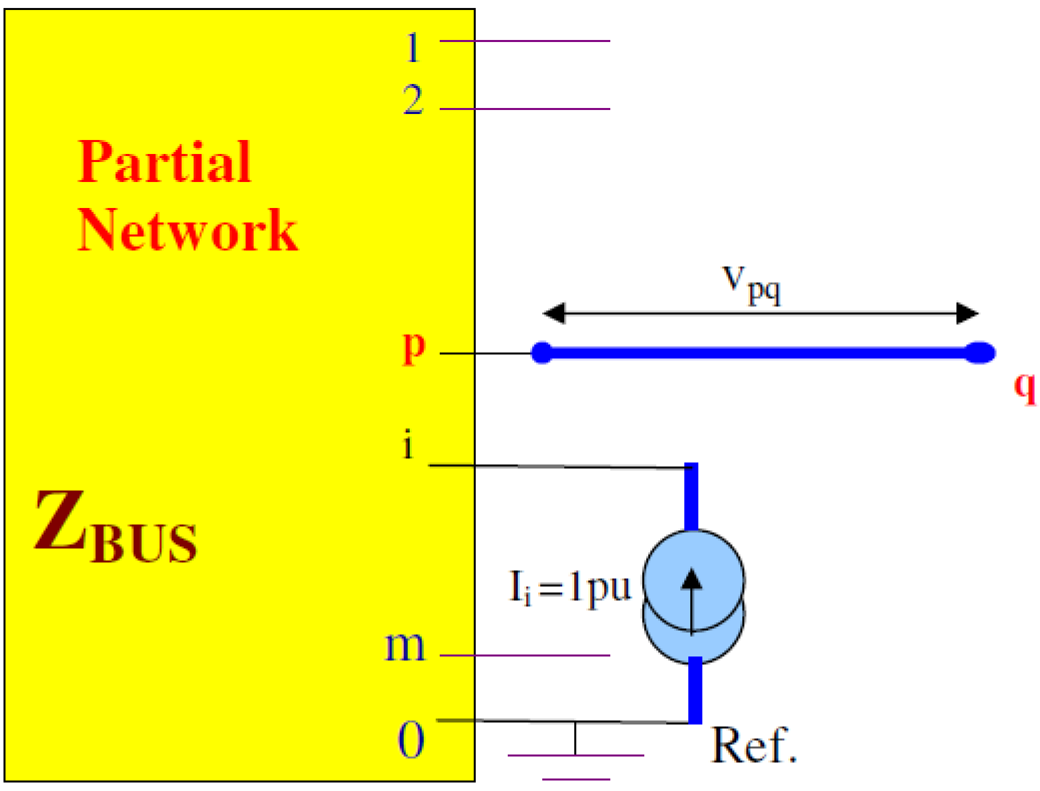


Fig.2 Calculation for Zqi

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2.....m; i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kq} I_q = Z_{kq}$ $\quad \forall$ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{pi} = 0 \qquad i = 1,2.....m : i \neq q$$

And $$Z_{pq} = 0.$$

Hence, from (18) (22) $$Z_{qi} = 0 \qquad i = 1,2......m; i \neq q$$

And $$Z_{qq} = z_{pq,pq} \qquad \backslash \quad (23)$$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \qquad (24)$$

2. Obtain the general expressions for Z_{bus} building algorithm when a link is added to the partial network. Dec 2016

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \qquad (25)$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector y_{pq-rs} is not equal to zero and $\mathbf{Z_{ij} = Z_{ji}}$ $\quad \forall$ i,j=1,2,…m,l. (26)

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki}\, I_i = Z_{ki} \qquad \forall \;\; k = 1, 2, \ldots i \ldots\ldots p, \ldots. m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

$$\text{So that } Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \;\; \forall \; i=1,2,\ldots i \ldots. p, \ldots q, \ldots. m, \neq l \tag{28}$$

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

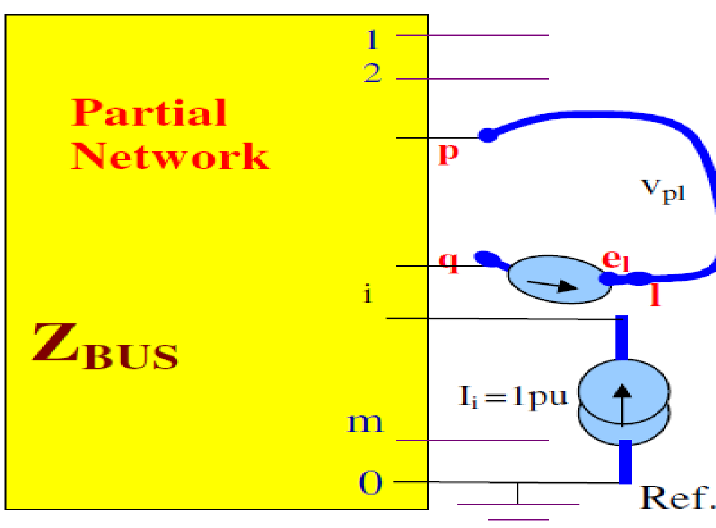


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs}\bar{v}_{rs} = 0 \qquad (30)$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \qquad (31)$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \qquad (32)$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2.....m; i \neq l \qquad (33)$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl} \quad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots q,\ldots m, l$ (34)

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl} \ \forall\ i=1,2,\ldots i\ldots p,\ldots q,\ldots m, \neq l$ (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \dfrac{\bar{y}_{pl,rs}\bar{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{36}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $\quad y_{pl,pl} = y_{pq,pq}$ (37)

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \quad (38)$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \quad (39)$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of *pqrsy*, are zero. Further, if *p* is not the reference node, then

$$Z_{li} = Z_{pi} - Z_{qi}$$

$$Z_{ll} = Z_{pl} - Z_{ql} - z_{pq,pq}$$
$$= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \quad (40)$$

3. **Prepare the Z_{bus} for the system shown using Z_{bus} building algorithmFor the positive sequence network data shown in table below, obtain ZBUS by building procedure.**

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

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Solution:

The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

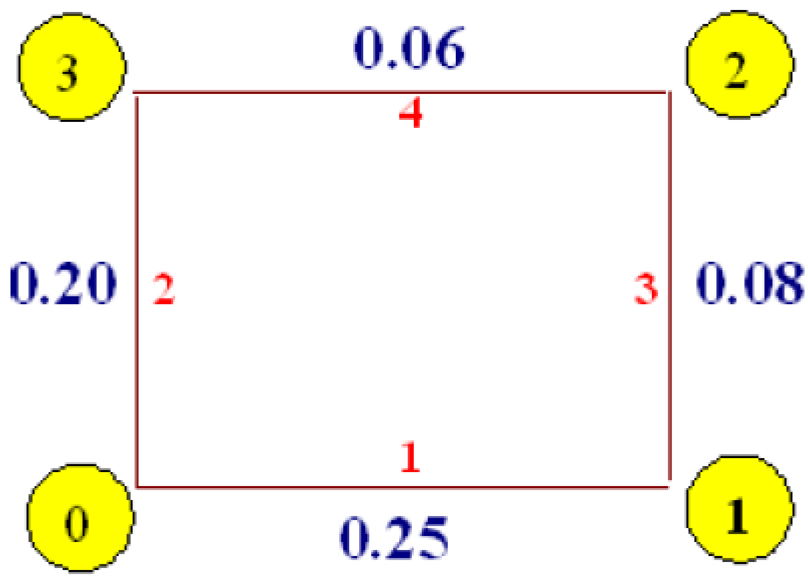
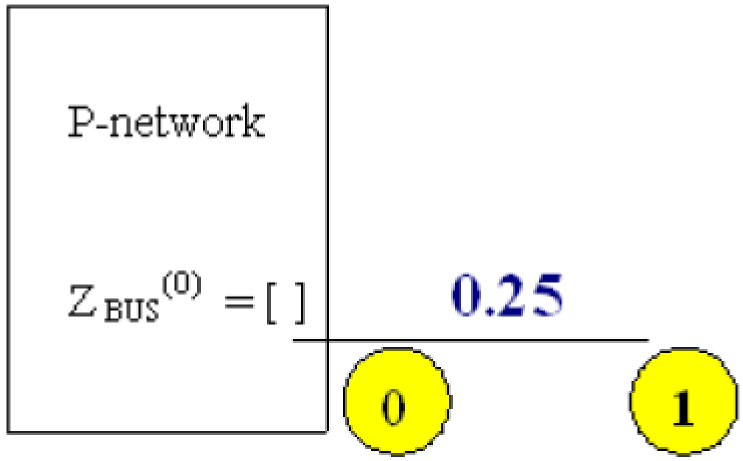


Fig. E1: Example System

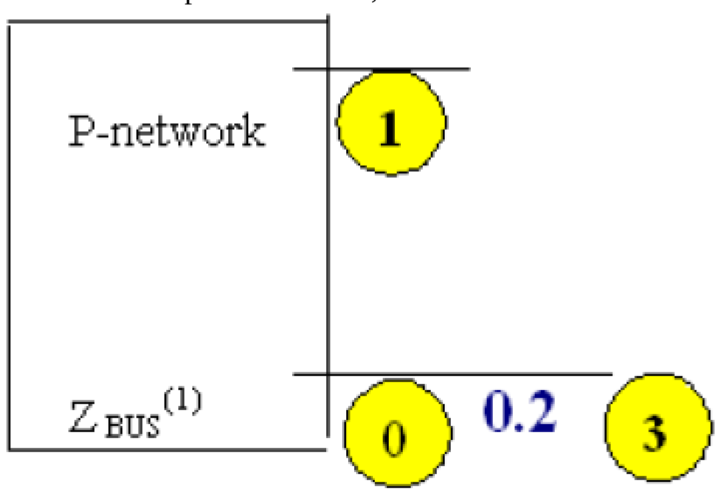
Consider building ZBUS as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



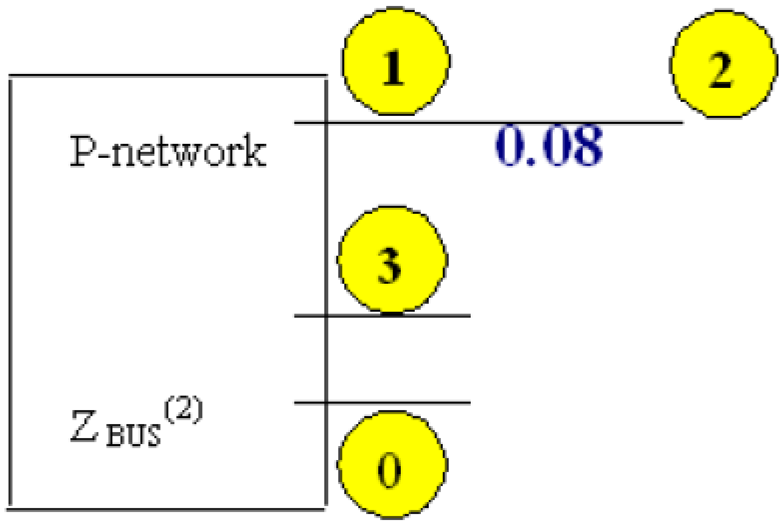
$$Z_{BUS}^{(1)} = \begin{array}{c|c|} & 1 \\ \hline 1 & 0.25 \\ \hline \end{array}$$

Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



$$Z_{BUS}^{(2)} = \begin{array}{c|c|c|} & 1 & 3 \\ \hline 1 & 0.25 & 0 \\ \hline 3 & 0 & 0.2 \\ \hline \end{array}$$

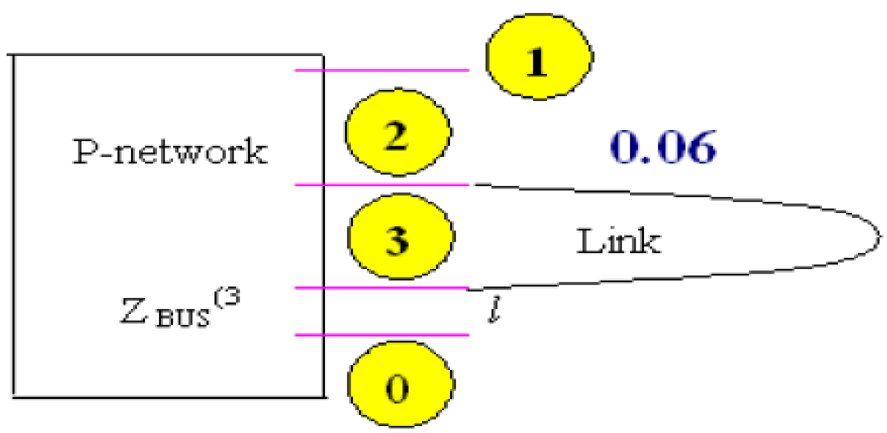
Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$Z_{BUS}^{(3)} =$

	1	3	2
1	0.25	0	0.25
3	0	0.2	0
2	0.25	0	0.33

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$Z_{BUS}^{(4)} =$

	1	3	2	l
1	0.25	0	0.25	0.25
3	0	0.2	0	-0.2
2	0.25	0	0.33	0.33
l	0.25	-0.2	0.33	0.59

The fictitious node *l* is eliminated further to arrive at the final impedance matrix as under:

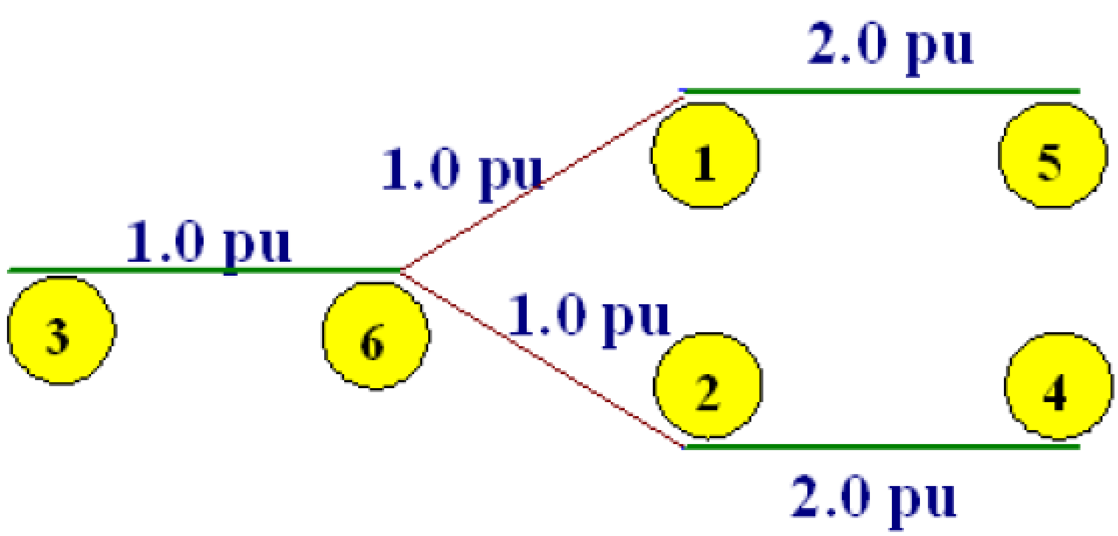
$Z_{BUS}^{(final)}$ =

	1	3	2
1	0.1441	0.0847	0.1100
3	0.0847	0.1322	0.1120
2	0.1100	0.1120	0.1454

Z_{BUS} =

	1	2	3	4	5
1	2	0	0	0	2
2	0	2	0	2	0
3	0	0	2	0	0
4	0	2	0	3	0
5	2	0	0	0	3

4. Prepare the Z_{bus} for the system shown using Z_{bus} building algorithm



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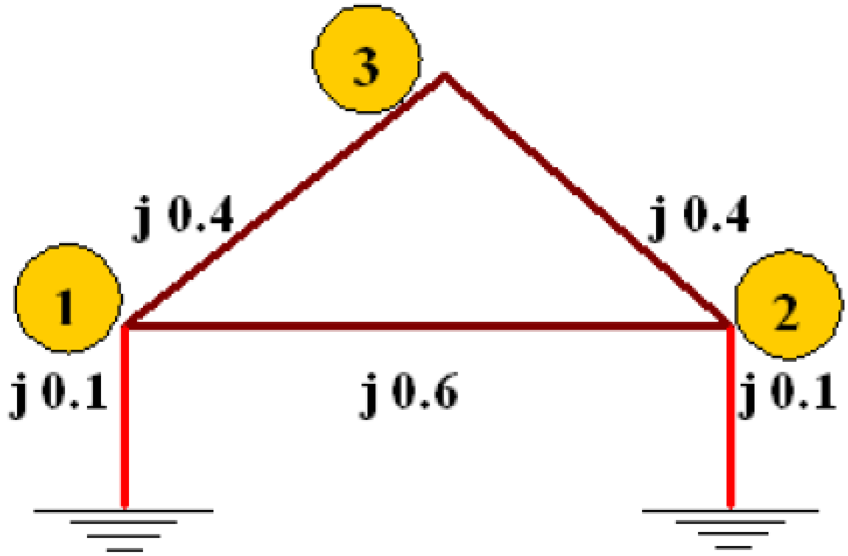
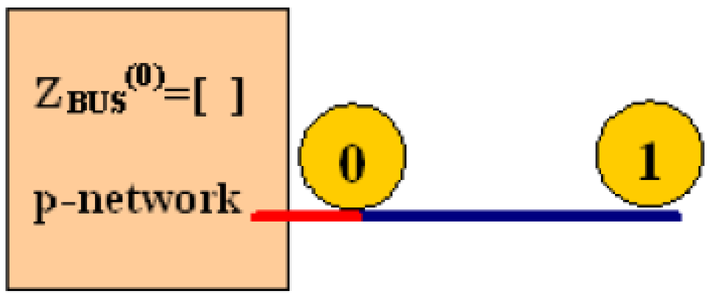


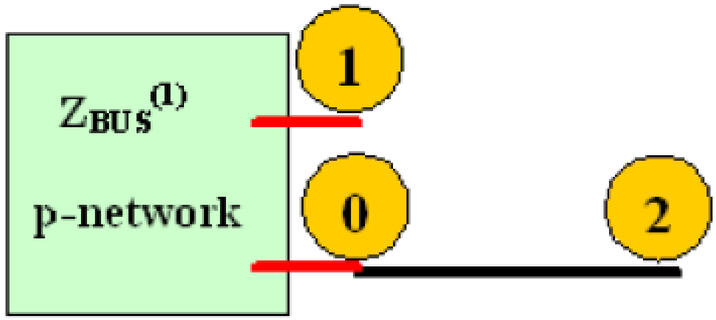
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



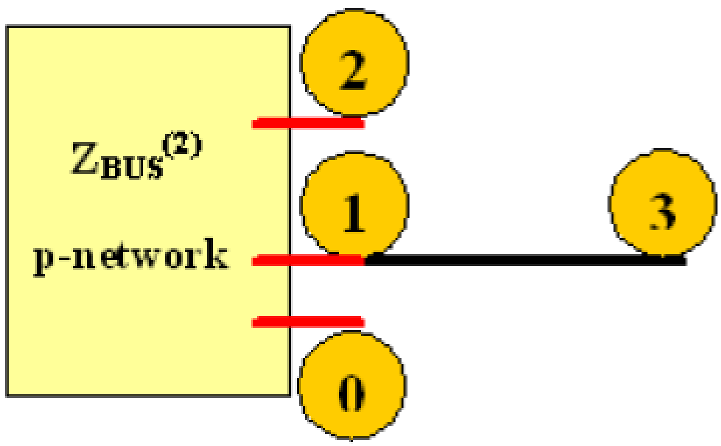
$$Z_{bus}^{(1)} = \begin{matrix} & 1 \\ 1 & [j0.1] \end{matrix}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



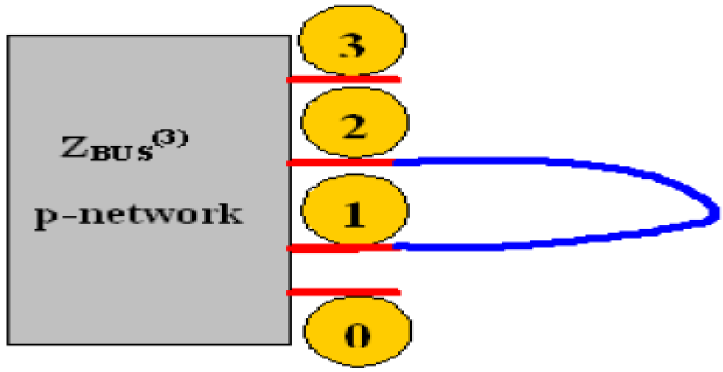
$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & \quad 2 \end{array} \\ \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & \quad 2 & \quad 3 \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{array}$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



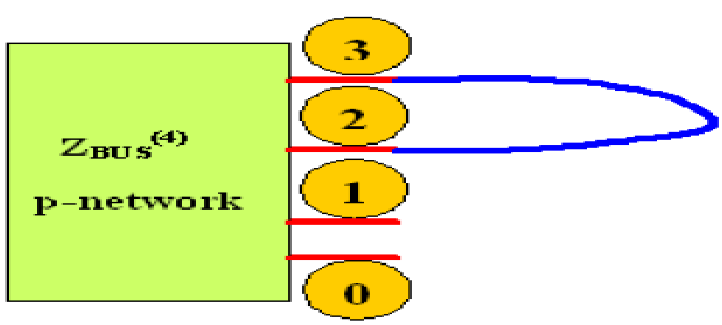
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix} \end{array}$$

Now the extra node-*l* has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} / Y_{nn}} \qquad \forall\ i,j = 1,2,3.$$

$$Z_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix} \end{array}$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 1 \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

5. Explain the formation of Z_{bus} using Z_{bus} building algorithm

Dec2016

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\bar{E}_{bus} = [Z_{bus}]\bar{I}_{bus} \tag{9}$$

When expanded so as to refer to an *n* bus system, (9) will be of the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + .Z_{1k}I_k \ldots + Z_{1n}I_n$$

$$\vdots$$

$$\vdots$$

$$E_k = Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots. + Z_{kn}I_n$$

$$\vdots$$

$$\vdots$$

$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots\ldots + Z_{nk}I_k + \ldots.. + Z_{nn}I_n \tag{10}$$

Now assume that the bus impedance matrix Zbus is known for a partial network of *m* buses and a known reference bus. Thus, Zbus of the partial network is of dimension *m*x*m*. If now a new element is added between buses *p* and *q* we have the following two possibilities:

(i) *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both *p* and *q* are buses existing in the partial network; in this case *p*-*q* is a **link** added to the p-network as shown in Fig 1b.

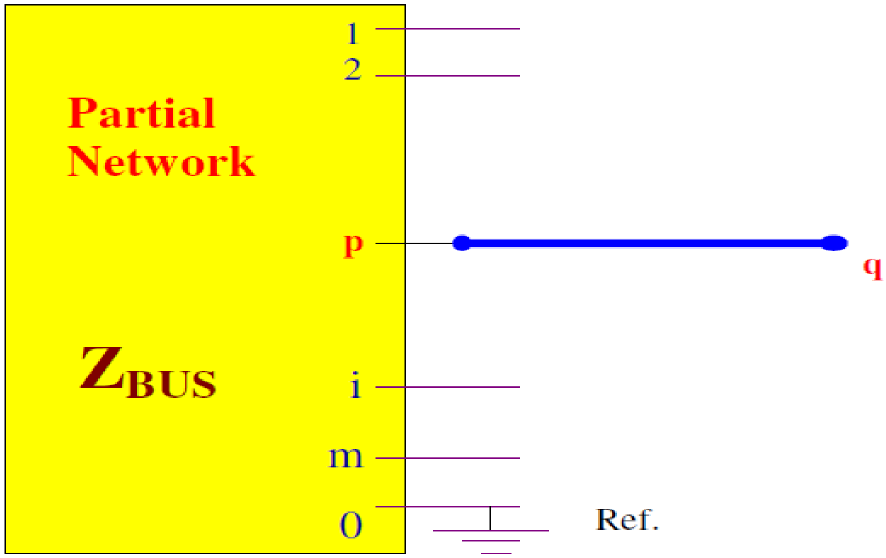


Fig 1a. Addition of branch p-q

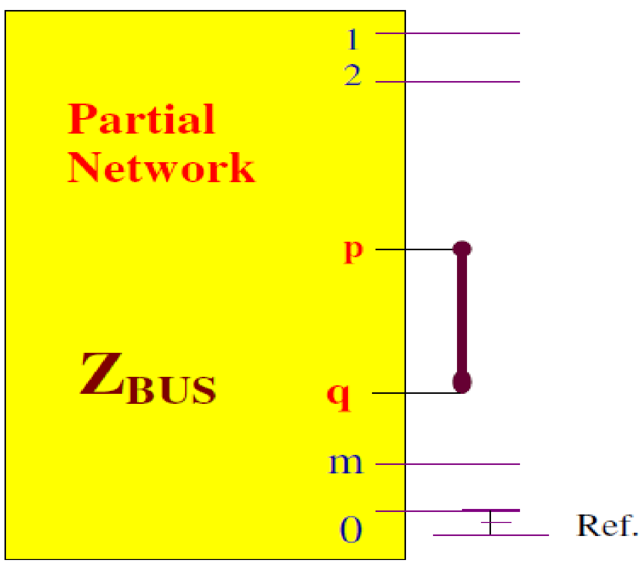


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix. If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

Module 2 & 3

1. Using generalized algorithm expressions for each case of analysis, explain the load flow studies procedure, as per the G-S method for power system having PQ and PV buses, with reactive power generations constraints.

June 2016, June 2015

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\ i = 2, 3, \ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any $(k+1)$*st*iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \quad \forall\ i=2,3,\ldots n \qquad \textbf{(18)}$$

Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current *(k+1)st*iteration. Hence these values are used. For buses *(i+1)…..n,* values from previous, *kth*iteration are used.

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall\ i = 2,3,\ldots n \qquad \textbf{(19)}$$

Where,e is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:
At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\operatorname{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij} V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\operatorname{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij} V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)

2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$

3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e (*k* +1) *i Q* computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st*iteration and the voltage is calculated with the value of Qi set as follows:

If $Q_i < Q_{i,min}$ Then $Q_i = Q_{i,min}$.

If $Q_i > Q_{i,max}$ Then $Q_i = Q_{i,max}$.

(23)

If in the subsequent iteration, if Qi falls within the limits, then the bus can be switched back to PV status.

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad \textbf{(24)}$$

whereα is a real number. When α =1, the value of (k +1) is the computed value. If 1<α<2 then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

2. Derive the expression in polar form for the typical diagonal elements of the sub matrices of the Jacobian in NR method of load flow analysis.

June 2017, Dec.2015, June 2015

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, \ldots x_n) = 0$, where $x_1, x_2, \ldots x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \qquad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \qquad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1\\k\neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1\\k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} = \begin{bmatrix}H & N\\ M & L\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \dfrac{\Delta|V|}{|V|}\end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

3. Compare NR and GS method for load flow analysis procedure in respect of the following i) Time per iteration ii) total solution time iii) acceleration factor iv)number of iterations

Dec 2016, Dec 2015, June 2017

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

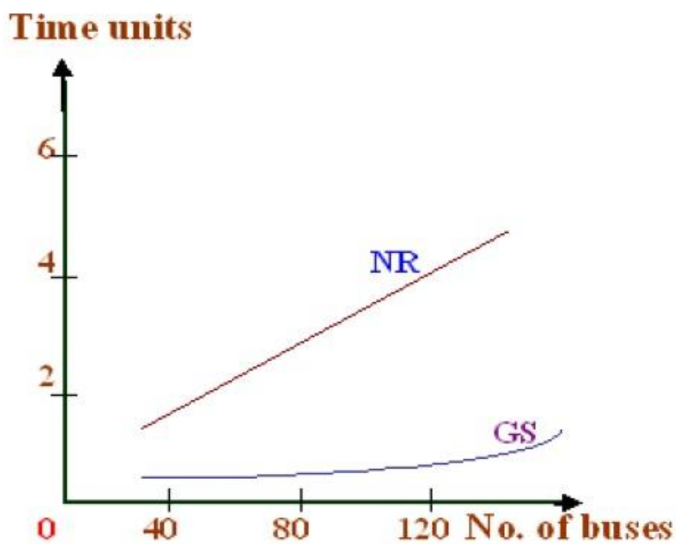


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

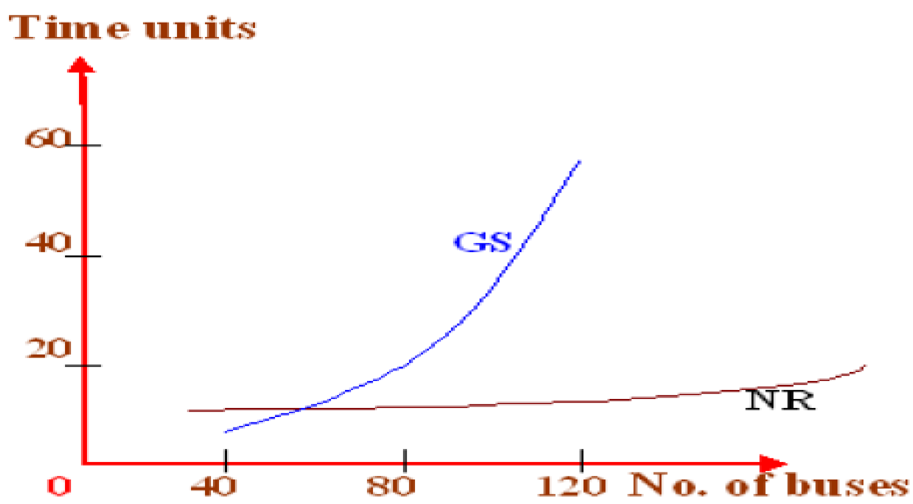


Figure 5. Total time of Iteration in GS and NR methods

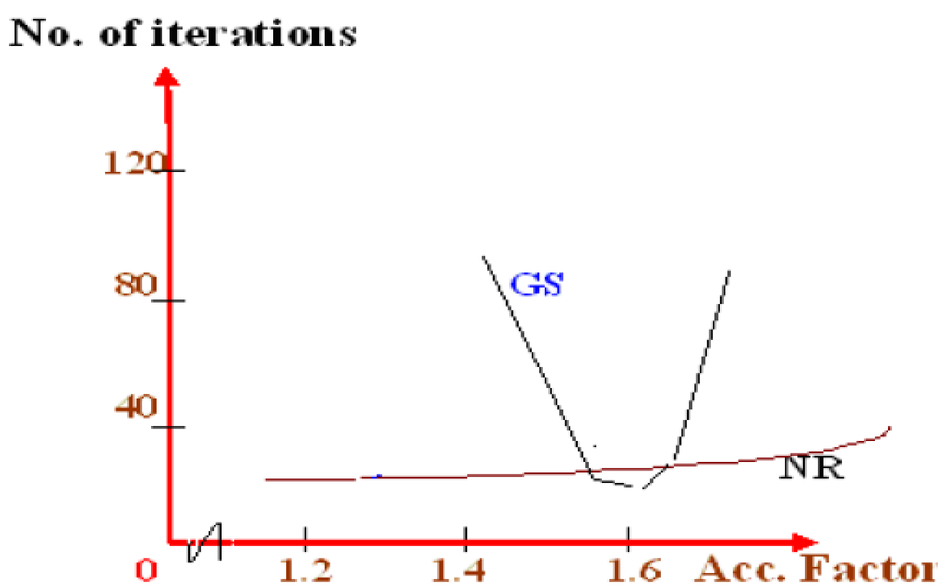


Figure 6. Influence of acceleration factor on load flow methods

The load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular. The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method.

These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a tradeoff between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

4. **Explain briefly fast decoupled load flow (FDLF) solution method for solving the non linear load flow equations.**

Dec 2015, June 2017

5. **What are the assumptions made in fast decoupled load flow method? Explain the algorithm briefly, through a flow chart.**

June 2015, Dec 2016

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $|V^{(r+1)}| = |V^{(r)}| + |\Delta V^{(r)}|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$

$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(38)}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \qquad \textbf{(39)}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

6. Explain the representation of transformer with fixed tap changing during the load flow studies

June 2017

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

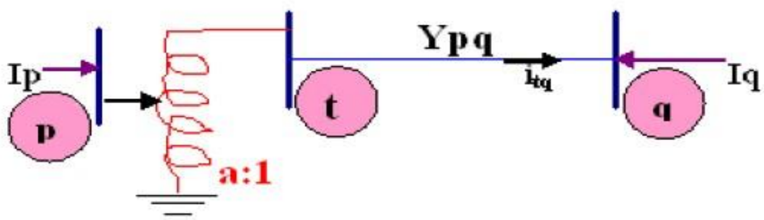


Fig. 2. Equivalent circuit of a tap setting transformer

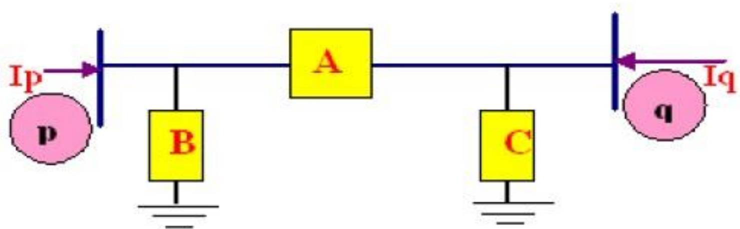


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

7. What is load flow analysis? What is the data required to conduct load flow analysis? Explain how buses are classified to carry out load flow analysis in power system. What is the significance of slack bus.

Dec 2015,Dec 2016, June 2017

Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system. Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load flow studies play a vital role in power system studies. Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

_ The Kirchhoff‟s relations holding good,
_ Capability limits of reactive power sources,
_ Tap-setting range of tap-changing transformers,
_ Specified power interchange between interconnected systems,
_ Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus:

The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 00 , as per flat-start procedure of iterative

solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of YBUS based LFA:

The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load flow analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of YBUS need to be stored for a n-bus system. Further, in the YBUS matrix, Yij = 0, if an incident element is not present in the system connecting the buses „i" and „j". since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, YBUS of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

Percentage sparsity of a given matrix of n^{th} order: $= \dfrac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$

$$S = (Z / n^2) \times 100 \ \% \qquad (1)$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of YBUS is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements YBUS can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While YBUS is thus highly sparse, it"s inverse, ZBUS, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \qquad (2)$$

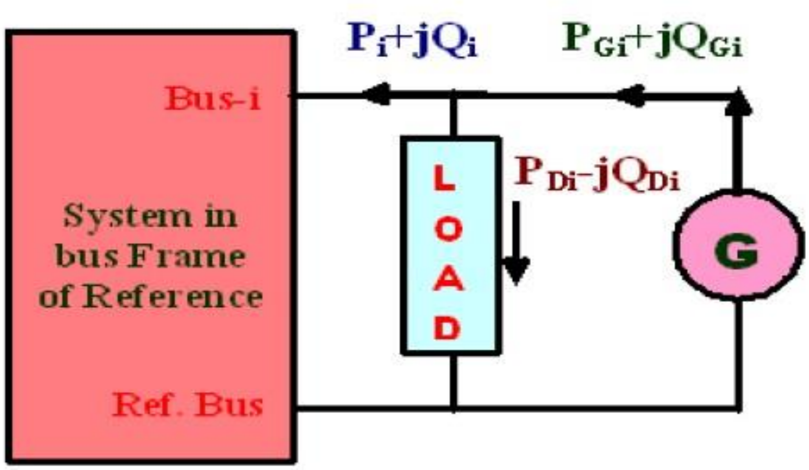


Fig.1 power flows at a bus-i

where Si = net complex power injected into bus *i*, SGi = complex power injected by the generator at bus *i*, and SDi = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

Si = _Sij " i = 1, 2,............. n

where Sij is the sum over all lines connected to the bus and *n* is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

Ii = IGi – IDi " i = 1, 2,...............n

where IGi is the current injected by the generator at the bus and IDi is the current drawn by the load (demand) at that bus. In the bus frame of reference

IBUS = YBUS VBUS

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix} \quad \text{is the vector of currents injected at the buses,}$$

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix} \quad \text{is the vector of complex bus voltages.}$$

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$S_i = V_i\ I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\ V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \qquad \textbf{(7)}$$

Let $\quad V_i \underline{\underline{\Delta}} |V_i| \angle\ \delta_i\ =\ |V_i| (\cos\ \delta_i\ +\ j \sin\ \delta_i\ )$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(\cos\delta_{ij} + j\sin\delta_{ij}\right)\left(G_{ij} - jB_{ij}\right) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\quad Y_{ij} = |Y_{ij}|\angle\theta_{ij} \qquad$ **(11)**

Again, we get from (7),

$$S_i = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\angle -\theta_{ij}\,|V_j|\angle -\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos\left(-\theta_{ij} + \delta_i - \delta_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\sin -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the „power flow equations" or the „load flow equations" in two alternative forms, corresponding to the n-bus system, where each bus-*i*is characterized by four variables, Pi, Qi, |Vi|, and di. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

8. Write a short note on i) acceleration factor in load flow solution.

June 2016

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right)$$

whereα is a real number. When α =1, the value of (*k* +1) is the computed value. If $1<\alpha<2$ then the value computed is extrapolated. Generally _ is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

9. For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.

Dec 2015,Dec 2016

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

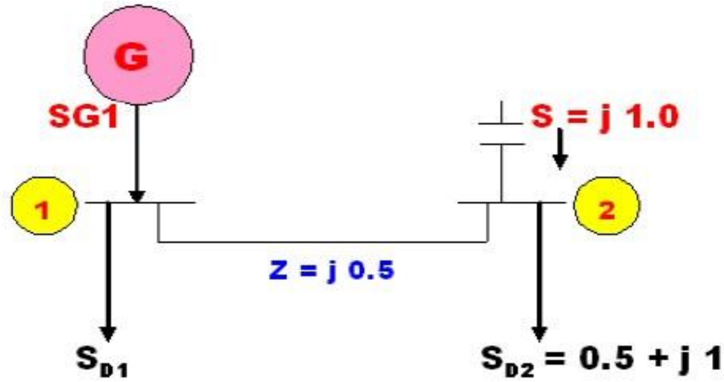


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^0 = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776\ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\ 0.23529\ = 0.970145\ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\ 0.249999\ = 0.970261\ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\ 0.248963 = 0.966237\ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\ 0.25\ \ = 0.966237\ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10-6 pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0 - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472 \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472 \angle 14.931^0$$

$$= 0.5 + j\,0.133329 \text{ pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

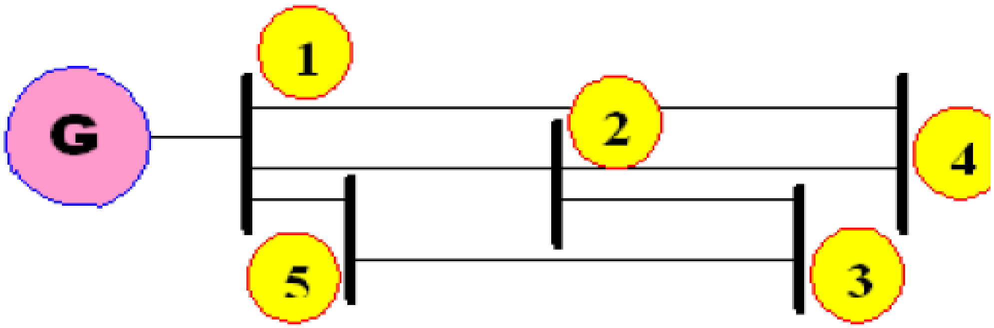
$$= 0.517472 \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -0.5 + j\,0.0 \text{ pu}$$

The total loss in the line is given by S12 + S21 = j 0.133329 pu Obviously, it is observed that there is no real power loss, since the line has no resistance.

10. For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.

June 2017



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{SP}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0^o
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

$$P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1$, $P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02 + j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294)\times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588)\times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294)\times 1.0\angle 0^0\}\right]$$

$$= 0.98140\angle -3.0665^o = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^o$; $\delta_2 = -3.0665^o$; $\delta_3 = 0^o$; $\delta_4 = 0^o$ and $\delta_5 = 0^o$

$\therefore\ \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^o\ (\delta_{ik} = \delta_i - \delta_k)$; $\delta_{32} = 3.0665^o$

$Q_3 = 1.04\ [1.02\ (0.0+j0.0) + 0.9814\ \{-1.17647 \times \sin(3.0665^o) - 4.70588 \times\cos(3.0665^o)\} + 1.04\{-9.41176 \times\cos(0^o)\} + 1.0\ \{0.0 + j0.0\} + 1.0\{-4.70588\times\cos(0^0)\}]$

$= 1.04\ [-4.6735 + 9.78823 - 4.70588] = 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \left\{(-1.7647 + j4.70588)\times(0.98140\angle -3.0665^{o})\right\}\right.$$
$$\left. - \left\{(-1.17647 + j4.70588)\times(1\angle 0^{o})\right\}\right]$$
$$= 1.05569\angle 3.077^{o} = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04\angle 3.077^{0}$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^{o} - Y_{42}V_2^{1} - Y_{43}V_3^{1} - Y_{45}V_5^{0}\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \left\{(-0.39215 + j1.56862)\times 1.02\angle 0^{o}\right\}\right.$$
$$\left. - \left\{(-0.58823 + j2.35294)\times(0.98140\angle -3.0665^{o})\right\}\right]$$
$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715\angle -7.303^{o} \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}V_1^{o} - Y_{52}V_2^{1} - Y_{53}V_3^{1} - Y_{54}V_4^{1}\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \left\{(-1.17647 + j4.70588)\times 1.02\angle 0^{o}\right\}\right.$$
$$\left. - \left\{(-1.17647 + j4.70588)\times 1.04\angle 3.077^{o}\right\}\right]$$
$$= 0.994618\angle -1.56^{o} = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02\angle 0^0$ pu $\qquad$ $V_2 = 0.98140\ \angle -3.066^0$ pu

$V_3 = 1.04\angle 3.077^0$ pu $\qquad$ $V_4 = 0.955715\angle -7.303^0$ pu

and $\qquad V_5 = 0.994618\angle -1.56^0$ pu

11. Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases
(i) All buses except bus 1 are PQ Buses
(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu
(i) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and 0.25_Q2_1.0 pu.

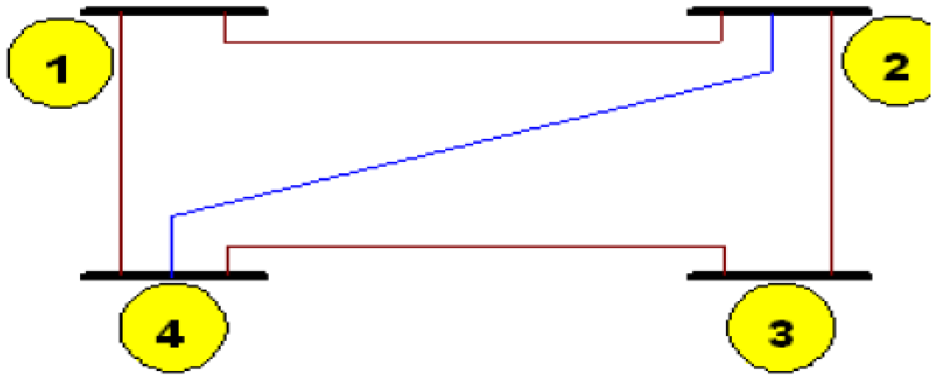


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	– 0.2	–
3	– 1.0	0.5	–
4	– 0.3	– 0.1	–

June 2015, June 2016

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$Y_{BUS} =$

3.0 – j9.0	–2.0 + j6.0	– 1.0 + j3.0	0
–2.0 + j6.0	3.666 – j11.0	– 0.666 + j2.0	– 1.0 + j3.0
–1.0 + j3.0	–0.666 + j2.0	3.666 – j11.0	–2.0 + j6.0
0	–1.0 + j3.0	– 2.0 + j6.0	3.0 – j9.0

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0\angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5+j0.2}{1.0-j0.0} - \{(-2.0+j6.0)\times(1.04\angle 0^o)\}\right.$$
$$\left. - \{(-0.666+j2.0)\times(1.0\angle 0^o)\} - \{(-1.0+j3.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.02014\angle 2.605^o$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0-j0.5}{1.0-j0.0} - \{(-1.0+j3.0)\times(1.04\angle 0.0^o)\}\right.$$
$$\left. - \{(-0.666+j2.0)\times(1.02014\angle 2.605^o)\} - \{(-2.0+j6.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.03108\angle -4.831^o$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}V_1^o - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3+j0.1}{1.0-j0.0} - \{(-1.0+j3.0)\times(1.02014\angle 2.605^o)\}\right.$$
$$\left. - \{(-2.0+j6.0)\times(1.03108\angle -4.831^o)\}\right]$$
$$= 1.02467\angle -0.51^o$$

Hence

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.02014\ \angle 2.605^0$ pu

$V_3^1 = 1.03108\ \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467\ \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\Big[|V_1|\,(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|\,(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22})$$
$$+ |V_3|\,(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|\,(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\Big]$$

$$= 1.04\,[1.04\,\{-6.0\} + 1.04\,\{11.0\} + 1.0\{-2.0\} + 1.0\,\{-3.0\} = 0.208 \text{ pu.}$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^0} - \{(-2.0 + j6.0)\times(1.04\angle 0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^o)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04\angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\angle 0^0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^o)\}\right.$$
$$\left. - \{(-0.666 + j2.0)\times(1.04\angle 1.846^o)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^0)\}\right]$$
$$= 1.035587\angle -4.951^0 \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^o)\}\right.$$
$$\left. - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^o)\}\right]$$
$$= 0.9985\angle -0.178^0$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

<u>Case (iii)</u>: Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$$V_1^1 = 1.04 \angle 0^0 \text{ pu} \qquad V_2^1 = 1.05645 \angle 1.849^0 \text{ pu}$$

$$V_3^1 = 1.038546 \angle -4.933^0 \text{ pu} \qquad V_4^1 = 1.081446 \angle 4.896^0 \text{ pu}$$

12. What are the advantages of Y-bus and Z-bus for load flow studies?

June 2015

The sparsity of the bus admittance matrixand the ease of building the bus admittance matrix, have made algorithms using theadmittance form of equations more popular.The most popular methods are the Gauss-Seidel method, the Newton-Raphsonmethod and the Fast Decoupled Load Flow method. These methods have beendiscussed in detail with illustrative examples. In smaller systems, the ease ofprogramming and the memory requirements, make GS method attractive. However,the computation time increases with increase in the size of the system. Hence, in largesystems NR and FDLF methods are more popular. There is a trade offbetweenvarious requirements like speed, storage, reliability, computation t ime, convergencecharacteristics etc. No single method has all the desirable features. However, NRmethod is most popular because of its versatility, reliability and accuracy.

Module 4

1. Derive the necessary condition for optimal operation of thermal power plants with the transmission losses considered. Jan.2014,June 2016

ECONOMIC DISPATCH INCLUDING TRANSMISSION LOSSES

When transmission distances are large, the transmission losses are a significant part of the generation and have to be considered in the generation schedule for economic operation. The mathematical formulation is now stated as

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_L}{\partial P_{Gi}}\right) = 0;\ i = 1\ldots\ldots n_g$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $$\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}},\ \text{(8.27) can be written as}$$

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant ***i***, $\mathbf{L}_i$. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i \,;\ i = 1\ldots\ldots n_g$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same, when losses are considered. A rigorous general expression for the loss PL is given by

$$P_L = \quad \Sigma_m \Sigma_n P_{Gm}\ B_{mn}\ P_{Gn}\ + \Sigma_n\ P_{Gn}\ B_{no} + B_{oo}$$

whereBmn, Bno , Boo called loss – coefficients , depend on the load composition. The assumption here is that the load varies linearly between maximum and minimum values. A simpler expression is

$$P_L = \quad \Sigma_m\ \Sigma_n\ P_{Gm}\ B_{mn}\ P_{Gn}$$

The expression assumes that all load currents vary together as a constant complex fraction of the total load current. Experiences with large systems has shown that the loss of accuracy is not significant if this approximation is used. An average set of loss coefficients may be used over the complete daily cycle in the coordination of incremental production costs and incremental transmission losses. In general, Bmn = Bnm and can be expanded for a two plant system as

$$P_L = B_{11}\ P_{G1}\ + 2\ B_{12}\ P_{G1}\ P_{G2} + B_{22}\ P_{G2}^{\ 2}$$

2. What are B- coefficients? Derive the matrix form of transmission loss equation.

June 2015, June 2017,Dec 2016, Dec 2015

DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference

(ii) The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

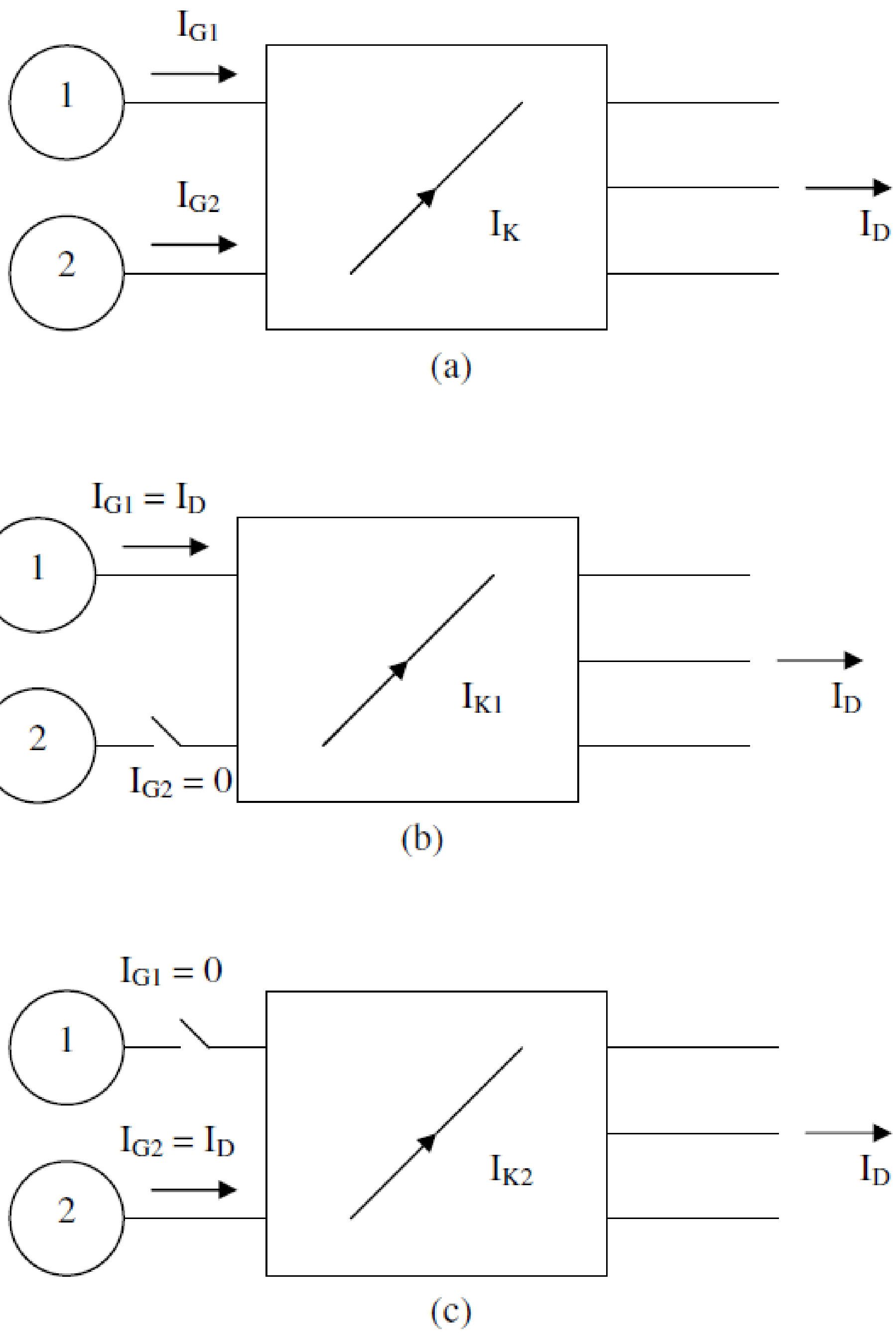


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1} I_{G1} + N_{K2} I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2.$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}{}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}{}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}{}^2|I_{G1}|^2 + N_{K2}{}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}{}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}{}^2 R_K$$

$$P_L = P_{G1}{}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}{}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}{}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}{}^2 + \ldots\ldots + \frac{P_{Gn}{}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}{}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_P \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

3. Explain problem formation and solution procedure of optimal scheduling for hydro thermal plants. **June 2016, Dec.2015**

4. What is the basic criterion for economical division of load between units within a plant?

June 2016, Dec.2015

ECONOMIC GENERATION SCHEDULING NEGLECTING LOSSES AND GENERATOR LIMITS

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand PD. Consider a system with **n**g number of generating plants supplying the total demand PD. If Fi is the cost of plant ***i***in Rs/h, the mathematical formulation of the problem of economic scheduling can be stated as follows:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

This is a constrained optimization problem, which can be solved by Lagrange‟s method.

LAGRANGE METHOD FOR SOLUTION OF ECONOMIC SCHEDULE

The problem is restated below:

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$£ = F_T + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial £}{\partial P_{Gi}} = 0 \text{ and } \frac{\partial £}{\partial \lambda} = 0$$

$$\frac{\partial £}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial £}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

The second equation is simply the original constraint of the problem. The cost of a plant Fi depends only on its own output PGi, hence

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \; ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

The above equation is called the co-ordination equation. Simply stated, for economic generation scheduling to meet a particular load demand, when transmission losses are neglected and generation limits are not imposed, all plants must operate at equal incremental production costs, subject to the constraint that the total generation be equal to the demand. From we have

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

It can be seen that l is dependent on the demand and the coefficients of the cost function.

5. Derive the necessary condition for optimal operation of thermal power plants without the transmission losses considered.

Jan 2014, June 2016, June 2015

ECONOMIC SCHEDULE INCLUDING LIMITS ON GENERATOR (NEGLECTING LOSSES)

The power output of any generator has a maximum value dependent on the rating of the generator. It also has a minimum limit set by stable boiler operation. The economic dispatch problem now is to schedule generation to minimize cost, subject to the equality constraint.

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \leq P_{Gi} \leq P_{Gi\,(max)}\,;\; i = 1, \ldots\ldots\ldots n_g$$

The procedure followed is same as before i.e. the plants are operated with equal incremental fuel costs, till their limits are not violated. As soon as a plant reaches the limit (maximum or minimum) its output is fixed at that point and is maintained a constant. The other plants are operated at equal incremental costs.

6. Draw and explain the following i) input-output curve ii) cost curve iii) incremental cost

The fuel cost in \$ / h for two 800 MW plants is given by

$F_1 = 400 + 6.0\, P_{G1} + 0.004\, P_{G1}^2$

$F_2 = 500 + b_2\, P_{G2} + c_2\, P_{G2}^2$

where P_{G1}, P_{G2} are in MW

(a) The incremental cost of power, λ is \$8 / MWh when total demand is 550MW. Determine optimal generation schedule neglecting losses.

(b) The incremental cost of power is \$10/MWh when total demand is 1300 MW. Determine optimal schedule neglecting losses.

(c) From (a) and (b) find the coefficients b_2 and c_2.

Solution:

a) $$P_{G1} = \frac{\lambda - b_1}{2c_1} = \frac{8.0 - 6.0}{2 \times 0.004} = 250 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 550 - 250 = 300 \text{ MW}$$

curve iv) Heat rate curve June 2016, June 2015

b) $$P_{G1} = \frac{\lambda - b_1}{2C_1} = \frac{10-6}{2\times 0.004} = 500 \text{ MW}$$

$$P_{G2} = P_D - P_{G1} = 1300 - 500 = 800 \text{ MW}$$

c) $$P_{G2} = \frac{\lambda - b_2}{2c_2}$$

From (a) $$300 = \frac{8.0 - b_2}{2c_2}$$

From (b) $$800 = \frac{10.0 - b_2}{2c_2}$$

Solving we get $b_2 = 6.8$

$c_2 = 0.002$

Module 5

1. **With the help of a flowchart, explain the method of finding the transient stability ofa given power system, based on Runge-Kutta method.**

June 2015, Dec.2015

Runge - Kutta method

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

$$\text{Given } \frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x(x_0, y_0, t_0)\, h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

$$l_1 = f_y(x_0, y_0, t_0)\, h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max} \sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max} \sin\delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{2}\right)}{M}\right] \Delta t$$

$$k_4 = (\omega_0 + l_3) \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin(\delta_0 + k_3)}{M}\right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6} [l_1 + 2l_2 + 2l_3 + l_4]$$

2. With the help of a flowchart, explain the method of finding the transient stability of a given power system, based on modified Euler's method.

June 2015, Dec 2016, Dec 2015, June 2017, June 2016

Modified Euler"s method:

Euler"s method is one of the easiest methods to program for solution of differential equations using a digital computer . It uses the Taylor"s series expansion, discarding all second–order and higher–order terms. Modified Euler"s algorithm uses the derivatives at the beginning of a time step, to predict the values of the dependent variables at the end of the step (t1 = t0 +Δt). Using the predicted values, the derivatives at the end of the interval are computed. The average of the two derivatives is used in updating the variables.

Consider two simultaneous differential equations:

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x0, y0, t0 at the beginning of a time step and a step size *h* we solve as follows:

Let

$$D_x = f_x(x_0, y_0, t_0) = \left.\frac{dx}{dt}\right|_0$$

$$D_y = f_y(x_0, y_0, t_0) = \left.\frac{dy}{dt}\right|_0$$

$$\left.\begin{aligned} x^P &= x_0 + D_x\, h \\ y^P &= y_0 + D_y\, h \end{aligned}\right\} \quad \text{Predicted values}$$

$$D_{xP} = \left.\frac{dx}{dt}\right|_P = f_x(x^P, y^P, t_1)$$

$$D_{yP} = \left.\frac{dy}{dt}\right|_P = f_y(x^P, y^P, t_1)$$

$$x_1 = x_o + \left(\frac{D_x + D_{xP}}{2}\right) h$$

$$y_1 = y_o + \left(\frac{D_y + D_{yP}}{2}\right) h$$

x_1 and y_1 are used in the next iteration. To solve the swing equation by Modified Euler‟s method, it is written as two first order differential equations:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{\max}\sin\delta}{M}$$

Starting from an initial value _o, _o at the beginning of any time step, and choosing a step size_t s, the equations to be solved in modified Euler‟s are as follows:

$$\left.\frac{d\delta}{dt}\right|_0 = D_1 = \omega_o$$

$$\left.\frac{d\omega}{dt}\right|_0 = D_2 = \frac{P_m - P_{max}\sin\delta_0}{M}$$

$$\delta^P = \delta_0 + D_1\,\Delta t$$

$$\omega^P = \omega_0 + D_2\,\Delta t$$

$$\left.\frac{d\delta}{dt}\right|_P = D_{1P} = \omega^P$$

$$\left.\frac{d\omega}{dt}\right|_P = D_{2P} = \frac{P_m - P_{max}\sin\delta^P}{M}$$

$$\delta_1 = \delta_0 + \left(\frac{D_1 + D_{1P}}{2}\right)\Delta t$$

$$\omega_1 = \omega_0 + \left(\frac{D_2 + D_{2P}}{2}\right)\Delta t$$

δ_1 and ω_1 are used as initial values for the successive time step. Numerical errors are introduced because of discarding higher–order terms in Taylor's expansion. Errors can be decreased by choosing smaller values of step size. Too small a step size, will increase computation, which can lead to large errors due to rounding off. The Runge- Kutta method which uses higher–order terms is more popular.

3. Explain the solution of swing equation by point-by-point method.

June 2017,June 2015,Dec 2016, Dec 2015

In this method the accelerating power during the interval is assumed constant at its value calculated for the middle of the interval.

The desired formula for computing the change in d during the n^{th} time interval is

$D\ d\ n = D\ d\ n\text{-}1 + [(D\ t)^2/M]\ Pa(n\text{-}1)$

where,

$D\ d\ n$ = change in angle during the n^{th} time interval

$D\ d\ n\text{-}1$ = change in angle during the $(n\text{-}1)^{th}$time interval

$D\ t$= length of time interval

Pa(n-1)= accelerating power at the beginning of the nthtime interval

Due attention is given to the effects of discontinuities in the accelerating power Pa which occur, for example, when a fault is applied or removed or when any switching operation takes place. If such a discontinuity occurs at the beginning of an interval, then the average of the values of Pa before and after the discontinuity must be considered. Thus, in computing the increment of angle occurring during first interval after a fault is applied at t=0, the above equation becomes:

$D\ d\ 1 = [(D\ t)^2/M]\ Pa0+/2$

where Pa0+ is the accelerating power immediately after the occurrence of the fault.

If the fault is cleared at the beginning of the m^{th}interval, then for this interval,

$Pa(m\text{-}1) = 0.5\ [Pa(m\text{-}1)^- + Pa(m\text{-}1)^+]$

Where $Pa(m\text{-}1)^-$ is the accelerating power before clearing and $Pa(m\text{-}1)^+$ is that immediately after clearing the fault.. If the discontinuity occurs at the middle of the interval, no special treatment is needed.